Composed Short Walsh's Sequences

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ABSTRACT

Walsh sequences (which formed an additive group and generated by Rademacher sequences) are used widely in the forward links of communication channels to mix the information on connecting to and at the backward links of these channels to sift through this information is transmitted to reach the receivers this information in correct form, especially in the pilot channels, the Sync channels, and the Traffics channel.

This research is useful to generate new sets of sequences (which are also additive groups) by compose Walsh sequences with the bigger lengths and the bigger minimum distance that assists to increase secrecy of this information and increase the possibility of correcting mistakes resulting in the channels of communication.

Index Terms

Walsh sequences, Additive group, Rademacher sequences, Orthogonal sequences. Autocorrelation function.

I. INTRODUCTION

In 1923, J.L. Walsh defined a system of orthogonal functions that is completely over the normalized interval (0,1). The method of specifying the Walsh functions of arbitrary order $N = 2^k$, k = 1,2,… had been a problem of considerable difficulty until the year 1970, when Byrnes and Swick showed that the Walsh functions could be obtained from Rademacher functions and from the solutions of certain differential equations. Byrnes and Swick considered the inherent symmetry properties of the Walsh functions of order N or as a set of functions denoted by $\{W_J(t), t \in (0,T), J = 0,1,\dots, N-1\}$ Walsh sequences of order 2^k, which are generated by the binary representation of Walsh functions of order $N = 2^k$, form a group under 2 addition (addition group). The set of these sequences except W_0 forms orthogonal set. Tables 1, 2, and 3 show the sequences of order 2^2, 2^3 and 2^4 respectively [1,2,3].

The Walsh functions can be generated by any of the following methods:

1. Using Rademacher functions.
2. Using Hadamard matrices.
3. Exploiting the symmetry properties of Walsh functions.
4. Using division ring under 2^k addition [4,5,6,7].

II. Research methods and materials

The Rademacher sequences of order $k = 3$ are :

R_0 = (0 0 0 0 0 0 0 0)

R_1 = (0 0 0 0 1 1 1 1)

R_2 = (0 0 1 1 0 0 1 1)

R_3 = (0 1 0 1 0 1 0 1)

The Rademacher sequences of order $k = 4$ are

R_1 = (0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1)

R_2 = (0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1)

R_3 = (0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1)

R_4 = (0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1)

Graphs of the Walsh functions of order 8 are given in Figure 1.

Tables:1, 2, 3 show sets of Walsh sequences of order $2^2 = 4$, $2^3 = 8$, and $2^4 = 16$ respectively.

Definition 1

Suppose $x = (x_0, x_1, \dots, x_{n-1})$ and

$y = (y_0, y_1, \dots, y_{n-1})$

are vectors of length n on $GF(2)=\{0,1\}$. The auto correlations function of x and y, denoted by $R_{x,y}$ is:

$$R_{x,y} = \sum_{i=0}^{n-1} (-1)^{x_i + y_i}$$

Where $x_i + y_i$ is computed mod 2. It is equal to the number of agreements components minus the number of disagreements corresponding to components. [3]

Definition 2

Suppose G is a set of binary vectors of length n

$$G = \{X; X = (x_0, x_1, \dots, x_{n-1}), x_i \in F_2 = \{0,1\}, i = \{0, \dots, n-1\}\}$$

Let $1^* = -1$ and $0^* = 1$. The set G is said to be orthogonal if the following two conditions are satisfied

$$\forall X \in G, \sum_{i=0}^{n-1} x_i^* \in \{-1,0,1\}, \quad or \quad |R_{x,0}| \le 1.$$

$$\forall X, Y \in G (X \ne Y), \sum_{i=0}^{n-1} x_i^* y_i^* \in \{-1,0,1\}, \quad or \quad |R_{x,y}| \le 1.$$

That is, the absolute value of "the number of agreements minus the number of disagreements" is equal to or less than 1. [8]

Definition 3

Hamming weight: The Hamming weight of binary vector $x = (x_0, x_1, \dots, x_{n-1})$ is the number of non zero components of x. [5]

Definition 4

Hamming distance $d(x,y)$: The Hamming distance between the binary vectors $x = (x_0, x_1, \dots, x_{n-1})$ and $y = (y_0, y_1, \dots, y_{n-1})$ is the number of the disagreements of the corresponding components of x and y.[5]

Definition 5

Minimum distance d: The minimum distance d of a set C of binary vectors is:

$$d = \min_{x,y \in C} d(x,y). \text{ [5]}$$

Definition 6

The code C of the form $[n, k, d]$ if each element (Codeword) has the: length n, The rank k is the number of information components (Message), minimum distance d. [5]

Definition 7

If W is a Walsh sequence and ω is any binary vector then:

$W(\omega)=\{w_i(\omega):w_i\in W\}$, we replace each " 1" in w_i by ω and each " 0 " in w_i by $\overline{\omega}$.[9]

III. RESULTS AND DISCUSSION

First Step

$W_4(W_4)$: We suppose the Walsh sequences of order 2^2 (Table 1) without w_0 that are:

w_1 = (0 0 1 1), w_2 = (0 1 1 0), w_3 = (0 1 0 1), the sequences define orthogonal set and code of the form C [n=4, k=2, d=2].

We will determine $W_4^*(W_4^*)$

We can see that:

$w_1(w_1) = (1100110000110011)$

$w_2(w_1) = (1100001100111100)$

$w_3(w_1) = (1100001111000011)$

If we write the matrix A such that their rows are: $R_i=w_i(w_1), i=1,2,3$ and theirs columns are: $C_i,\ j=1,2,...,16$, after that we remove: all zero columns and all equals columns except one, we have the matrix:

$$\overline{A}=\begin{bmatrix}1&1&0&0&1&0&1\\1&0&1&0&1&1&0\\1&0&1&1&0&0&1\end{bmatrix}$$

In this matrix if we consider rows $r_i : i=1,2,3$ and columns $c_j : j=1,2,3,...,7$ we can denote:

- $c_1=c_2+...+c_7$
- $\overline{c}_2=c_3, \overline{c}_4=c_5, \overline{c}_6=c_7$

We consider the matrix ${}_2\overline{G}_2=\overline{A}$ the rank of the matrix ${}_2\overline{G}_2$ is 3

$${}_2\overline{G}_2=\begin{bmatrix}1&\boxed{1}&0&\boxed{0}&1&\boxed{0}&1\\1&\boxed{0}&1&\boxed{0}&1&\boxed{1}&0\\1&\boxed{0}&1&\boxed{1}&0&\boxed{0}&1\end{bmatrix}$$

Since the rank of the matrix ${}_2\overline{G}_2$ is 3 then the ranks of the matrices $\overline{A}$ and A are also 3. We determine the other non zero elements of $Span\{r_1,r_2,r_3\}$ we get:

$r_4=r_1+r_2=(0\ 1\ 1\ 0\ 0\ 1\ 1)$

$r_5=r_1+r_3=(0\ 1\ 1\ 1\ 1\ 0\ 0)$

$r_6=r_2+r_3=(0\ 1\ 1\ 1\ 1\ 0\ 0)$

$r_7=r_1+r_2+r_3=(1\ 1\ 0\ 1\ 0\ 1\ 0)$

The set $\{r_1,r_2,...,r_7\}$ is closed under the addition, orthogonal sequences and determine code of the form ζ[n=7,k=3,d=4],with the corresponding generator matrix :

$${}_2G_2(w_1)=\begin{bmatrix}w_1(w_1)\\w_2(w_1)\\w_3(w_1)\end{bmatrix}=\begin{bmatrix}1&1&0&0&1&\boxed{1}&0&0&0&\boxed{0}&1&1&\boxed{0}&0&1&1\\1&1&0&0&0&\boxed{0}&1&1&0&\boxed{0}&1&1&\boxed{1}&1&0&0\\1&1&0&0&0&\boxed{0}&1&1&1&\boxed{1}&0&0&\boxed{0}&0&1&1\end{bmatrix}$$

This matrix has the rank 3 then we can generate by this matrix the new Code:

$R_1=w_1(w_1)=(110\ 0110000110011)$
$R_2=w_2(w_1)=(110\ 0001100111100)$
$R_3=w_3(w_1)=(110\ 0001111000011)$
$R_4=R_1+R_2=(0000111100\ 001111)$
$R_5=R_1+R_3=(0000111111110000)$
$R_6=R_2+R_3=(0000000011111111)$
$R_7=R_1+R_2+R_3=(1100110011001100)$
$R_7=w_0(w_1)$

The set C = $\{R_1,R_2,R_3,R_4,R_5,R_6,R_7\}$ is a closed set under the addition and an orthogonal sequences set of the form ξ [n=16, k=3, d=8].

a. By the same way each of ${}_2\overline{G}_2(w_2)$, ${}_2\overline{G}_2(w_3)$, ${}_2\overline{G}_2(\overline{w}_1)$, ${}_2\overline{G}_2(\overline{w}_2)$, and ${}_2\overline{G}_2(\overline{w}_3)$ a closed sets under the addition and each of them is an orthogonal sequences set of the form [n = 7, k = 3, d = 4].

b. By the same way each of ${}_{2}G_{2}(w_2)$, ${}_{2}G_{2}(w_3)$, ${}_{2}G_{2}(\overline{w}_1)$, ${}_{2}G_{2}(\overline{w}_2)$, and ${}_{2}G_{2}(\overline{w}_3)$ is a closed set under the addition and each of them is an orthogonal set of the form [n =16,k=3,d=8].

*The result is 6 sets closed under the addition and each of them contains 7 Orthogonal sequences with the: length n = 7, rank k = 3, and distance d = 4.

* The result is 6sets closed under the addition and each of them contains 7 Orthogonal sequences with the: length n = 16, rank k = 3, and distance d = 8.

Second step

$W_8(W_4)$: We suppose the Walsh sequences of order 2^3 (Table 2) without ω_0 that are:

The sequences define an orthogonal set and code of the form C [n = 8, k = 3, d = 4].

We determine:

$\omega_1(w_1) =$(1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1)

$\omega_2(w_1) =$(1 1 0 0 1 1 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1 1 0 0 1 1 0 0)

$\omega_3(w_1) =$(1 1 0 0 1 1 0 0 0 0 1 1 0 0 1 1 1 1 0 0 1 1 0 0 0 0 1 1 0 0 1 1)

$\omega_4(w_1) =$(1 1 0 0 0 0 1 1 0 0 1 1 1 10 0 1 1 0 0 0 0 1 1 0 0 1 1 1 1 0 0)

$\omega_5(w_1) =$(1 1 0 0 0 0 1 1 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 1 1 0 0 0 0 1 1)

$\omega_6(w_1) =$(1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0)

$\omega_7(w_1) =$(1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1)

If we write the matrix A such that their rows are: $R_i = \omega_i(w_1), i = 1,2,...,7$ and theirs columns are: $C_i,\ j = 1,2,...,32$, after that we remove: all zero columns and all equals columns except one, we have the matrix:

$$\overline{A} = \begin{bmatrix} 1&1&0&1&0&1&0&0&1&0&1&0&1&0&1 \\ 1&1&0&0&1&0&1&0&1&0&1&1&0&1&0 \\ 1&1&0&0&1&0&1&1&0&1&0&0&1&0&1 \\ 1&0&1&0&1&1&0&1&0&0&1&0&1&1&0 \\ 1&0&1&1&1&1&0&0&1&1&0&1&0&0&1 \\ 1&0&1&1&0&0&1&0&1&1&0&0&1&1&0 \\ 1&0&1&1&0&0&1&1&0&0&1&1&0&0&1 \end{bmatrix}$$

In this matrix if we consider their rows $r_i : i = 1,2,3,...,7$ and their columns $c_j : j = 1,2,3,...,15$ we can denote:

- $c_1 = c_2 + ... + c_{15}$
- $\overline{c}_2 = c_3, \overline{c}_4 = c_5, ..., \overline{c}_{14} = c_{15}$
- $r_5 = r_2 + r_3 + r_4$
- $r_6 = r_1 + r_3 + r_4$
- $r_7 = r_1 + r_2 + r_4$

The rank of the matrix:

$${}_{3}\overline{G}_{2} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 1&1&\boxed{0}&\boxed{1}&0&1&0&0&1&\boxed{0}&1&\boxed{0}&1&0&1 \\ 1&1&\boxed{0}&\boxed{0}&1&0&1&0&1&\boxed{0}&1&\boxed{1}&0&1&0 \\ 1&1&\boxed{0}&\boxed{0}&1&0&1&1&0&\boxed{1}&0&\boxed{0}&1&0&1 \\ 1&0&\boxed{1}&\boxed{0}&1&1&0&1&0&\boxed{0}&1&\boxed{0}&1&1&0 \end{bmatrix}$$

Is 4 then the ranks of the matrices $\overline{A}$ and A are also 4.

We determine the other non zero elements of $Span\{r_1, r_2, r_3, r_4\}$ we get:

$r_8 = r_1 + r_2 =$(0 0 0 1 1 1 1 0 0 0 0 1 1 1 1)

$r_9 = r_1 + r_3 =$(0 0 0 1 1 1 1 1 1 1 1 0 0 0 0)

$r_{10} = r_1 + r_4 =$(0 1 1 1 1 0 0 1 1 0 0 0 0 1 1)

$r_{11} = r_2 + r_3 =$(0 0 0 0 0 0 0 1 1 1 1 1 1 1 1)

$r_{12} = r_2 + r_4 =$(0 1 1 0 0 1 1 1 1 0 0 1 1 0 0)

$r_{13} = r_3 + r_4 =$(0 1 1 0 0 1 1 0 0 1 1 0 0 1 1)

$r_{14} = r_1 + r_2 + r_3 =$ (1 1 0 1 0 1 0 1 0 1 0 1 0 1 0)

$r_{15} = r_1 + r_2 + r_3 + r_4 =$ (0 1 1 1 1 0 0 0 0 1 1 1 1 0 0)

And the corresponding rows in the matrix A are:

$R_8 =$(0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1)

$R_9 =$(0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 10 0 0 0 0 0 0 0)

$R_{10} =$(0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1)

$R_{11} =$(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1)

$R_{12} =$(0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0)

$R_{13} =$(0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1)

$R_{14} =$(1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0)

$R_{15} =$(0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0)

The set $\{r_1, r_2, ..., r_{15}\}$ is closed under the addition, orthogonal sequences and determine code of the form ζ[*n* =15, *k* =4, *d* = 8],And the generator matrix :

$${}_{3}G_{2} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}$$

Corresponding to the matrix ${}_{3}\overline{G}_{2}$ generates another set $\{R_1, R_2, ..., R_{15}\}$ that is closed under the addition, and each of them contains 15 orthogonal sequences and is of the form:

ξ [*N* = 32, *k* = 4, *d* = 16].

a. By the same way each of ${}_{3}\overline{G}_{2}(w_2)$,…, ${}_{3}\overline{G}_{2}(w_7)$, ${}_{3}\overline{G}_{2}(\overline{w}_1)$, …, ${}_{3}\overline{G}_{2}(\overline{w}_7)$, and ${}_{3}\overline{G}_{2}(\overline{w}_7)$ closed sets under the addition and each of them contains 15 orthogonal sequences of the form [*n* =15, *k* = 4, *d* = 8].

b. By the same way each of ${}_{3}G_{2}(w_2)$,…, ${}_{3}G_{2}(w_7)$, ${}_{3}G_{2}(\overline{w}_1)$,…, ${}_{3}G_{2}(\overline{w}_6)$, and ${}_{3}G_{2}(\overline{w}_7)$ closed sets under the addition and each of them contains 15 orthogonal sequences of the form [*n* =32, *k* = 4, *d* = 16].

* The result is 15sets closed under the addition and each of them contains 15 Orthogonal sequences with the: length *n* =15, rank *k* = 4, and distanced = 8.

* The result is 15 sets closed under the addition and each of them contains 15 Orthogonal sequences with the: length *n* =32, rank *k* = 4, and distance d = 16.

Third step

$W_4(W_8)$ We denote that:

$R_1 = w_1(\omega_1) =$(1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1)

$R_2 = w_1(\omega_1) =$(1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0)

$R_2 = w_1(\omega_1) =$(1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1)

If we write the matrix A such that: their rows are: $R_i = w_i(\omega_1), i = 1,2,3$ and theirs

The columns are:, after that we remove: all zero columns and all equals columns except one, we have the matrix:

$$\overline{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} = \overline{A} \quad \text{in the first step}$$

In this matrix if we consider their rows $r_i : i = 1,2,3$ and their columns

$c_j : j = 1,2,3,...,7$ We can denote:

- $c_1 = c_2 + ... + c_7$
- $\overline{c}_2 = c_3, \overline{c}_4 = c_5, \overline{c}_6 = c_7$

The rank of the matrix:

$${}_{2}\overline{G}_{3} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} = {}_{2}\overline{G}_{2}$$

is 3 then the ranks of the matrices $\overline{A}$ and A are also 3.

We determine the other non zero elements of $Span\{r_1, r_2, r_3\}$ we get:

$r_4 = r_1 + r_2 =$(0 1 1 0 0 1 1)

$r_5 = r_1 + r_3 =$(0 1 1 1 1 0 0)

$r_6 = r_2 + r_3 = (0\ 0\ 0\ 1\ 1\ 1\ 1)$

$r_7 = r_1 + r_2 + r_3 = (\ 1\ 1\ 0\ 1\ 0\ 1\ 0)$

And the corresponding rows in the matrix A are:

$R_4 =$(0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1)

$R_5 =$(0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0)

$R_6 =$(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1)

$R_7 =$ (1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0)

The set $\{r_1, r_2, ..., r_7\}$ is closed under the addition, orthogonal sequences and determine code of the form ζ [n=7, k=3, d=4], And the generator matrix:

$$ {}_2G_3 = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} $$

Corresponding to the matrix ${}_2\overline{G}_3$ generates another set $\{R_1, R_2, ..., R7\}$ that is closed under the addition and contains 7 orthogonal sequences of the form:

ξ [N = 32, k = 3, d =16].

a. In the same way each of ${}_2G_3(\omega_2), ..., {}_2G_3(\omega_7)$, ${}_2\overline{G}_3(\overline{w}_1)$, ${}_2\overline{G}_3(\overline{\omega}_2)$, …, and ${}_2\overline{G}_3(\overline{\omega}_7)$ closed sets under the addition and each of them is an orthogonal sequences set of the form [n =7, k =3, d = 4].

b. In the same way each of ${}_2\overline{G}_3(\omega_2), ..., {}_2\overline{G}_3(\omega_7)$, ${}_2G_3(\overline{\omega}_1)$, ${}_2G_3(\overline{\omega}_2)$, …,and ${}_2G_3(\overline{\omega}_7)$ closed sets under the addition and each of them is an orthogonal set of the form: [n =32, k =3, d =16].

* The result is 14sets, closed under the addition, and each contains 7 Orthogonal sequences with the: length n = 7, rank k = 3, and distance d = 4.

* The result is 14sets, closed under the addition, and each contains 7 Orthogonal sequences with the: length n = 32, rank k = 3, and distance d = 16.

Forth step

$W_8(W_8)$: We denote that:

$\omega_1(\omega_1) =$
(11110000111100001111000011110000000011110000111100 00111100001111)

$\omega_2(\omega_1) =$
(11110000111100000000111100001111000011110000111111 11100001111000)

$\omega_3(\omega_1) =$
(11110000111100000000111100001111111100001111000000 00111100001111)

$\omega_4(\omega_1) =$
(11110000000011110000111111110000111100000000111100 00111111110000)

$\omega_5(\omega_1) =$
(11110000000011110000111111110000000011111111000011 11000000001111)

$\omega_6(\omega_1) =$
(11110000000011111111000000001111000011111111000000 00111111110000)

$\omega_7(\omega_1) =$
(11110000000011111111000000001111111100000000111111 11000000001111)

If we write the matrix A that their rows are: $R_i = \omega_i(w_1), i = 1,2,...,7$ and theirs

The columns are:, after that we remove: all zero columns and all equals columns except one, we have the matrix:

$$ \overline{A} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} $$

$= \overline{A}$ In the second step

In this matrix if we consider their rows $r_i : i = 1,2,3,...,7$ and their columns

$c_j : j = 1,2,3,...,15$ We can denote:

- $c_1 = c_2 + ... + c_{15}$
- $\bar{c}_2 = c_3, \bar{c}_4 = c_5, ..., \bar{c}_{14} = c_{15}$
- $r_5 = r_2 + r_3 + r_4$
- $r_6 = r_1 + r_3 + r_4$
- $r_7 = r_1 + r_2 + r_4$

The rank of the matrix:

$$\overline{G}_{3\;3} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \boxed{0} & \boxed{1} & 0 & 1 & 0 & 0 & 1 & \boxed{0} & 1 & \boxed{0} & 1 & 0 & 1 \\ 1 & 1 & \boxed{0} & \boxed{0} & 1 & 0 & 1 & 0 & 1 & \boxed{0} & 1 & \boxed{1} & 0 & 1 & 0 \\ 1 & 1 & \boxed{0} & \boxed{0} & 1 & 0 & 1 & 1 & 0 & \boxed{1} & 0 & \boxed{0} & 1 & 0 & 1 \\ 1 & 0 & \boxed{1} & \boxed{0} & 1 & 1 & 0 & 1 & 0 & \boxed{0} & 1 & \boxed{0} & 1 & 1 & 0 \end{bmatrix} = \overline{G}_{3\;2}$$

Is 4 then the ranks of the matrices $\overline{A}$ and A are also 4.

We determine the other non zero elements of $Span\{r_1, r_2, r_3, r_4\}$ and the corresponding rows in the matrix *A*.

The set $\{r_1, r_2, ..., r_{15}\}$ is closed under the addition and contains 15 orthogonal sequences and determine code of the form ζ[*n* =15, *k* = 4, *d* = 8],And the generator matrix :

$$G_{3\;3} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}$$

Corresponding to the matrix $\overline{G}_{3\;3}$ generates another set $\{R_1, R_2, ..., R_{15}\}$ that is closed under the addition and contains also 15orthogonal sequences and of the form:

ξ [*n* = 64, *k* = 4, *d* = 32].

a. In the same way each of $\overline{G}_{3\;3}(\omega_2), ..., \overline{G}_{3\;3}(\omega_7)$, $\overline{G}_{3\;3}(\overline{\omega}_1), ..., \overline{G}_{3\;3}(\overline{\omega}_6)$, and $\overline{G}_{3\;3}(\overline{\omega}_7)$ closed sets under the addition and each of them is an orthogonal sequences set of the form [*n*=15,*k*=4,*d*=8].

b. In the same way each of $G_{3\;3}(\omega_2), ..., G_{3\;3}(\omega_7)$, $G_{3\;2}(\overline{\omega}_1)$, ..., $G_{3\;3}(\overline{\omega}_6)$ and $G_{3\;3}(\overline{\omega}_7)$ closed sets under the addition and each of them is an orthogonal set of the form [*n* = 64, *k* = 4, *d* =32].

* The result is 14 sets closed under the addition and contains 15 orthogonal sequences with the: length *n* =15, rank *k*=4, and distance d = 8.

* The result is 14 sets closed under the addition and contains 15 orthogonal sequences with the: length *n* = 64, rank *k* = 4, and distance *d* =32.

IV. CONCLUSION

1. The formation 6 sets closed under the addition and each of them contains 7 orthogonal sequences of the form *n* = 7, *k* = 3, *d* = 4.
2. The formation 6 sets closed under the addition and each of them contains 7 orthogonal sequences of the form *n* = 16, *k* = 3, *d* = 8.
3. The formation 14sets closed under the addition and each of them contains 15 orthogonal sequences of the form *n* = 15, *k* = 4, *d* = 8.
4. The formation 14sets closed under the addition and each of them contains 15 orthogonal sequences of the form *n* = 32, *k* = 4, *d* = 16.
5. The formation 14 sets closed under the addition and each of them contains 7 orthogonal sequences of the form *n* = 7, *k* = 3, *d* = 4
6. The formation 14sets closed under the addition and each of them contains 7 orthogonal sequences of the form *n* = 32, *k* = 3, *d* = 16.
7. The formation 14sets closed under the addition and each of them contains 15 orthogonal sequences of the form *n* = 15, *k* = 4, *d* = 8.
8. The formation 14sets closed under the addition and each of them contains 15 orthogonal sequences of the form *n* = 64, *k* = 4, *d* = 32.

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VII. Graphs and tables

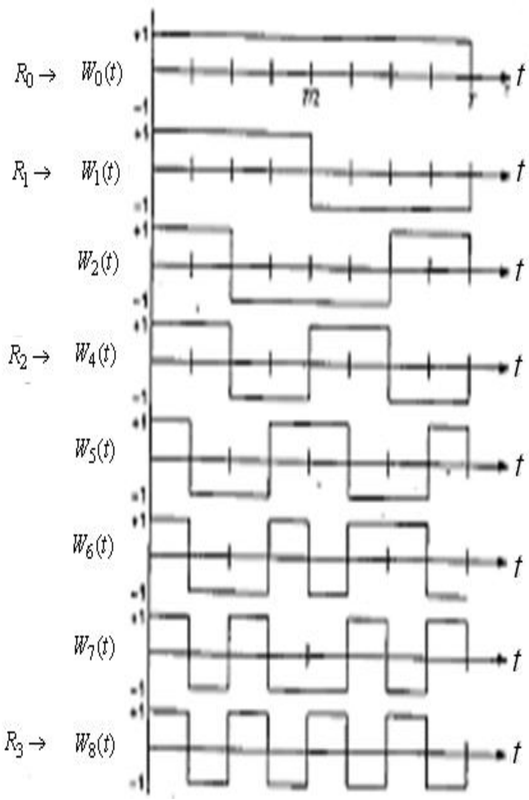

Figure 1. Walsh functions of order 8= 2^3

Index Sequences	Walsh Sequences of order 4=2^2
00	w_0=0000
01	w_1=0011
10	w_2=0110
11	w_3=0101

Table 1. Walsh sequences of order 4 = 2^2

Index Sequences	Walsh Sequences of order 8=2^3
000	w_0 = 0 0 0 0 0 0 0 0
001	w_1 = 0 0 0 0 1 1 1 1
010	w_2 = 0 0 1 1 1 1 0 0
011	w_3 = 0 0 1 1 0 0 1 1
100	w_4 = 0 1 1 0 0 1 1 0
101	w_5 = 0 1 1 0 1 0 0 1
110	w_6 = 0 1 0 1 1 0 1 0
111	w_7 = 0 1 0 1 0 1 0 1

Table 2. Walsh sequences of order 8=2^3